Chapter 5

ENERGY BALANCE MODEL OF EVAPORATION: EXPERIMENTS 1 AND 2

In the present study there were four fluxes which controlled the water balance in the field - sprinkler irrigation and precipitation which added water to the field, and evaporation and deep drainage which removed water. Irrigation was easily measured at any point using catch cans, and precipitation was considered to be uniform. Of the other two fluxes, deep drainage is notoriously difficult to measure with any degree of precision. With microlysimeters, evaporation is more easily measured but this method proved to be quite difficult and time consuming on the first day after irrigation, at least with our silty clay loam soil. Also, ML's are known to deviate from true evaporation since the field soil dries by both drainage and evporation while ML's dry only by evaporation.

The energy balance model (EBM) of evaporation estimation requires only two measurements of soil surface temperature at any point in the field, one at the time of maximum and one at the time of minimum temperature. Since these measurements may be easily and rapidly made with a hand-held infrared thermometer this method seemed ideal for the evaporation measurements at many points in the field necessary for the completion of the water balance study. However, the studies

by Ben Asher et al. (1983, 1984) showed some potential problems with the method.

Experiment 2 was conceived to run concurrently with Experiment 1 with the ML measurements of evaporation serving as a direct comparison to the energy balance estimates of evaporation resulting from infrared thermometer measurements of ML surface temperatures. The IR thermometer measurements needed for the EBM were taken from the soil surface inside the ML's so that evaporation estimates from the EBM would reflect conditions within the ML's. Thus there should be a 1:1 correlation between evaporation estimates from the two methods if the EBM works correctly. Reference dry soil temperatures needed for the EBM were taken from soil packed in a plastic container (34 cm deep by 29 cm diameter) buried in the field so that soil surfaces inside and outside the container were at the same elevation.

This chapter will present the results of this comparison between the microlysimeter and energy balance methods. The first 2 sections present the theory of the EBM as given by Ben-Asher et al. (1983), and show the result of application of the EBM to Experiment 2 data. Comparison of EBM estimates of evaporation to ML data showed the EBM to overestimate by about 100% on average. The third section presents a discussion of the assumptions used in developing the EBM with an eye towards identifying those assumptions most likely to cause inaccuracy.

Chapter 6 deals with improvements to the EBM.

Energy Balance Theory.

The model of Ben-Asher et al. (1983) was based on analysis of the surface energy balances of a dry soil and of a drying soil. The main assumption used was that the energy balances differed significantly only in latent heat flux, there being no latent heat flux from dry soil. Some equations from Ben-Asher et al. are included here since later work will build on their theory.

The energy balances for a dry and a drying soil are (denoting dry by subscript o, and drying by subscript d):

$$R_{no} = H_o + G_o$$
 [5-1]

$$R_{nd} = H_d + G_d + \mathbf{L}_e E$$
 [5-2]

where R_n is the net radiation, H is the sensible heat flux, G is the soil heat flux, and $\mathbf{L_e}E$ is the latent heat flux (all in W m⁻²). R_n for the two soils is:

$$R_{no} = K_{in} (1 - \alpha_o) + L_{in} - L_{o,out}$$
 [5-3]

$$R_{nd} = K_{in} (1 - \alpha_d) + L_{in} - L_{d,out}$$
 [5-4]

where K_{in} is the solar (shortwave) radiation (W m⁻²), α is the albedo, and L is the long wave radiation (W m⁻²) with subscripts 'in' and 'out' indicating incoming and outgoing

long wave radiation.

Subtracting the energy balance equations and including the net radiation equations, an equation for latent heat flux is:

$$\mathbf{L_e} E = (G_o - G_d) + K_{in} (\alpha_o - \alpha_d) + (H_o - H_d) + (L_{o,out} - L_{d,out})$$
 [5-5]

Integration gives the total evaporative flux over the period t_1 to t_2 :

$$\int \mathbf{L_e} E \ dt = \int [(G_o - G_d) + K_{in} (\alpha_o - \alpha_d) + (H_o - H_d) + (L_{o,out} - L_{d,out})] \ dt$$
 [5-6]

Given the simplifying assmumption that:

$$\int [(G_o - G_d) + K_{in}(\alpha_o - \alpha_d)] dt << \int \mathbf{L_e} E dt$$
 [5-7]

the latent heat flux can be written:

$$\int \mathbf{L}_{e} \mathbf{E} \ dt = \int \left[(\mathbf{H}_{o} - \mathbf{H}_{d}) + (\mathbf{L}_{o,out} - \mathbf{L}_{d,out}) \right] \ dt$$
 [5-8]

where only the sensible and outgoing longwave heat fluxes need be determined. Although Fox (1968) showed the plausibility of Equation 5-7 the validity of this assumption will be examined later.

The sensible heat fluxes for dry and drying soils may be written as (Rosenberg et al. 1983, p.124):

$$H_o = \rho C_p (T_o - T_a) / r$$
 [5-9]

$$H_{d} = \rho C_{p} (T_{d} - T_{a}) / r$$
 [5-10]

where ρ is the air density (1.21 kg m⁻³), C_p is the specific heat of air (1005 J kg⁻¹ °K⁻¹), r is the aerodynamic resistance to heat transport (s m⁻¹), T_o and T_d are the surface temperatures (°C) of the dry and drying soils, respectively, and T_a is the temperature of the air (°C) at a height above the ground specified in the definition of aerodynamic resistance. Ben-Asher et al. (1983), citing Rosenberg et al. (1974) used for r:

$$r = 126 \text{ U}^{-0.96}$$
 [5-11]

where U is wind speed in m s⁻¹. With the assumptions that 1) the air temperature at reference height (1 m) is everywhere the same, and 2) the aerodynamic resistance to heat flux is everywhere the same, the equations for sensible heat flux may be subtracted:

$$H_o - H_d = \rho C_p (T_o - T_d) / r$$
 [5-12]

thus eliminating the need to measure air temperature.

Outgoing long wave radiation is described by the Stefan-Boltzmann law:

$$L_{out} = \varepsilon \sigma T^4$$
 [5-13]

where σ is the Stefan-Boltzmann constant (5.67E-08 W m⁻² °K⁻¹), ε is the emissivity (taken to be an average of 0.95), and T is the surface temperature (°K). Re-writing the longwave radiation term in Equation 5-8 we have:

$$L_{o,out} - L_{d,out} = \varepsilon \sigma (T_o^4 - T_d^4)$$
 [5-14]

Examining Equations 5-8, 5-12 and 5-14 it is clear that only $T_{\rm o}$, $T_{\rm d}$ and wind speed need be measured in order to calculate latent heat flux on an instantaneous basis. However, instantaneous measurement of even these 3 variables is onerous, as is the next best approximation which would be to measure the variables on a sufficiently short time step to allow accurate numerical integration over the period of interest. These measurements become particularly difficult if many sites are to be measured. Accordingly, Ben-Asher et al. assumed that soil surface temperature could be approximated by a sine function of time:

$$T(t) = \overline{T} + 0.5(T_{max} - T_{min}) \sin(wt)$$
 [5-15]

where $\overline{T}=(T_{max}+T_{min})/2$ is the average temperature, 0.5($T_{max}-T_{min}$) is the amplitude, and $w=2\pi/\tau$ is the angular frequency (radians per unit time). Also, T_{max} is the maximum temperature and T_{min} the minimum temperature in the period, and τ is the period (units of time). For Equation 5-15, t is time in the

same units as τ with t=0 corresponding to the time when T(0) $= \overline{T} \quad \text{and } T \text{ is increasing (i.e. start of sine wave).}$

In order to avoid sine functions to the 4th power, it is necessary to reduce the 4th order temperature terms in equation 15 to first order terms. Letting $\Delta T = T_{\circ} - T_{d}$ and $\overline{T}_{m} = (T_{\circ} + T_{d})/2$ we have:

$$T_o^4 - T_d^4 = (\bar{T}_m + \Delta T/2)^4 - (\bar{T}_m - \Delta T/2)^4$$
 [5-16]

or

$$T_0^4 - T_d^4 = 4\bar{T}_m^3 \Delta T + \bar{T}_m(\Delta T)^3$$
 [5-17]

Thus Equation 5-14 is equivalent to:

$$L_{o,out} - L_{d,out} = \sigma \epsilon 4 \bar{T}_m^3 \Delta T [1 + (\Delta T)^2 / (4 \bar{T}_m^2)]$$
 [5-18]

and the approximation:

$$L_{\text{g,out}} - L_{\text{d,out}} \doteq 4\varepsilon\sigma\overline{T}_{\text{m}}^{3}(T_{\text{o}} - T_{\text{d}})$$
 [5-19]

has an error of $\varepsilon\sigma(\Delta T)^3\overline{T}_m$. The difference between Equations 5-14 and 5-19, summed over one half day with 15 minute time steps is only about 0.01% (See Appendix B for computer program and results).

Introducing Equations 5-12 and 5-19 into Equation 5-8 and re-arranging we have:

$$\int \mathbf{L_e} \mathbf{E} \ dt = \int \left[\rho C_p / r + 4 \epsilon \sigma \overline{T}_m^3 \right] (T_o - T_d) dt$$
 [5-20]

Typical field measurements reveal that the minimum temperatures $T_{\text{o,min}}$ and $T_{\text{d,min}}$ are almost equal. If Equation 5-15 is used to represent T_{o} and T_{d} this fact results in (Ben-Asher et al. 1983, Eq. 12):

$$T_{o} - T_{d} = 0.5 (T_{o,max} - T_{d,max}) + 0.5 (T_{o,max} - T_{d,max}) \sin(wt)$$

$$= 0.5 (T_{o,max} - T_{d,max}) (1 + \sin(wt))$$
 [5-21]

Replacing T_{\circ} - T_{d} in Equation 5-20 with Equation 5-21, we have:

or:

$$\int \mathbf{L_e} \mathbf{E} \ dt = \int [\rho C_p / r + 4\varepsilon \sigma \overline{T}_m^3]$$

$$[0.5 (T_{o,max} - T_{d,max}) (1 + \sin(wt))] \ dt \qquad [5-23]$$

Ben-Asher et al. (1983, Eq. 15a) defined the period, τ , as 24 h, and assumed that the wind speed over the period of integration was constant so that r was constant. They also assumed the quantity \overline{T}_m^3 to be essentially constant over the range of \overline{T}_m with $\overline{T}_m^3 = (\overline{T}_o - \overline{T}_d)/2$ where \overline{T}_o and \overline{T}_d are the diurnal average soil surface temperatures of dry and drying soils, respectively. Invoking these assumptions, they integrated Equation 5-23 from -3 hours to 9 hours resulting in:

$$\int \mathbf{L_e} E \ dt = 6 \left[\rho C_p / r + 4 e \sigma \overline{T}_m^3 \right] \left[1 + (2/\pi)^{0.5} \right]$$

$$(T_{o,max} - T_{d,max}) \qquad [5-24]$$

The limits of integration were chosen by assuming that all energy flux terms would be in phase, by noting that the soil heat flux is positive from -3 h to 9 h given that Equation 5-15 correctly describes the soil surface temperature over time, and by assuming (implied) that $\mathbf{L_e}\mathbf{E}$ is positive only when G is positive, and that negative values of $\mathbf{L_e}\mathbf{E}$ could be ignored. Note that S in Equation 1-2 is, by Equation 5-24,:

$$S = 8.70 \left[\rho C_p / r + 4 \epsilon \sigma \overline{T}_m^3\right]$$
 [5-25]

Use of Equation 5-24 requires only 3 measurements; daily average daytime wind speed, maximum reference dry soil surface temperature, and maximum drying soil surface temperature. The first two measurements may be made conveniently at one site and the third may be made at as many sites as desired.

Application of Model.

Taking $\mathbf{L}_{\mathbf{e}}$ as constant, the energy balance model is:

$$\int E dt = 8.70 \left[\rho C_p/r + 4\varepsilon \sigma \overline{T}_m^3\right] \left(T_{o,max} - T_{d,max}\right) / \mathbf{L_e}$$
 [5-26]

Equation 5-26 was used to predict daily evaporation, $E_{\rm est}$, (mm) and the estimates were regressed against actual, $E_{\rm a}$, (mm) as measured by weighing the ML's (3rd day after irrigation omitted due to lost wind speed data). The resulting regression equation shows that Equation 5-26 over-predicts evaporation by an average factor of about 2 (Figure 5-1A):

$$E_a = -0.309 + 0.549 E_{est}$$
 [5-27]

The R^2 value, at 0.49, was lower than that obtained for the regression of E_a against the midday temperature depression, $(T_{o,max} - T_{d,max})$. This indicates that the model assumptions, implicit in the R.H.S. of Equation 5-26, are suspect. However, both the slope and intercept terms in Equation 5-27 were significant at the 10% level.

Regression of E_a against E_{est} with dummy variables for wall type resulted in equations for steel and for plastic ML's (Figure 5-1A, R^2 = 0.52):

$$E_a = 0.06 + 0.45 E_{est}$$
, Plastic [5-28]

$$E_a = -0.92 + 0.70 E_{est}$$
, Steel [5-29]

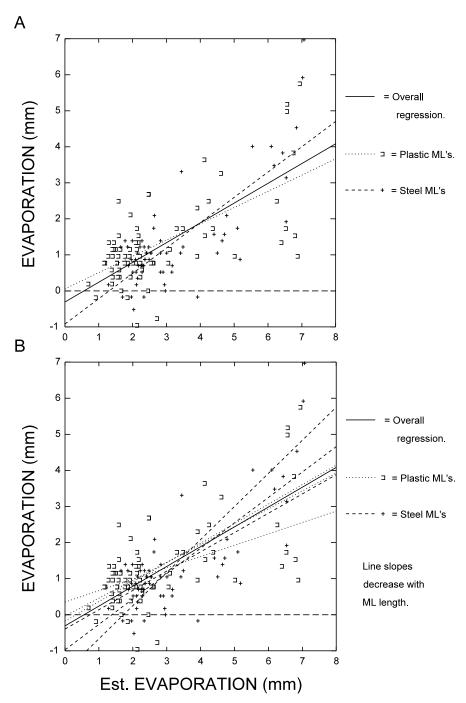


Figure 5-1. A. Regression of ML evaporation vs. predictions from the energy balance model of Equation 5-26 showing differences due to wall material. $R^2=0.49$ for overall regression, improved to 0.52 when dummy variables for wall material were included. B. Regression on same data using dummy variables to separate treatments showing differences due to wall material and length. $R^2=0.55$.

Most striking is the negative intercept of almost 1 mm for steel ML's indicating a possible non-linearity between Equation 5-26 and the evaporation measured by steel ML's. By contrast, the intercept for plastic ML's is nearly zero.

A third regression of E_a against $E_{\rm est}$ included dummy variables for the treatment effects on intercept and slope following the model presented in Appendix E (Equation E13) but with the quantity $(T_{\rm o,max} - T_{\rm d,max})$ replaced with $E_{\rm est}$ and E replaced with E_a . Six regression lines resulted (Table 5-1, Figure 5-1B). Steel ML's all showed more negative intercepts than did plastic. Slopes for the 30 and 20 cm steel ML's were higher than those for 30 and 20 cm plastic ML's.

The R^2 value of 0.55 was slightly lower than the R^2 of 0.57 for the regression of E_a against ($T_{o,max}$ - $T_{d,max}$) with dummy variables. This result was surprising, even considering that one day's data were omitted from the analysis of E_{est} vs. E_a . The inclusion of wind and average surface temperature effects in Equation 5-26 should have improved its performance over the quantity ($T_{o,max}$ - $T_{d,max}$) as a predictor of evaporation but such was not the case. In fact, when the 3rd day after irrigation was omitted from the regression of E_a against ($T_{o,max}$ - $T_{d,max}$) with dummy variables, the resulting R^2 value was 0.60 showing that, with the same data set, the quantity ($T_{o,max}$ - $T_{d,max}$) was more highly correlated with evaporation than was E_{est} from

Table 5-1.

E_{est}

Regression analyses for daily evaporation, E_a , (mm) with the estimated evaporation, $E_{\rm est}$, (mm) from Equation 5-26 as the independent variable; and dummy variables for length and wall type treatments.

Model: $E_a = b_0 + b_1 E_{est}$ $r^2 = 0.494$, n = 136. significance parameter estimate std. error -0.309 intercept 0.165 0.063 0.549 0.048 0.000

Model: $E_a = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_5x_5 + b_6E_{est}$ $+ b_{16}x_{16} + b_{26}x_{26} + b_{36}x_{36} + b_{46}x_{46} + b_{56}x_{56}$

See Appendix E for explanation of model.

$r^2 = 0.546$, parameter	n = 136 estimate	std. error	significance	
intercept	-0.035 -0.362 0.387 -0.939 -0.139 -1.549 0.495 0.040 -0.180 0.209 0.045 0.421	0.451 0.622 0.557 0.621 0.555 0.711 0.131 0.180 0.168 0.176 0.164 0.193	0.938 0.561 0.488 0.133 0.803 0.031 0.000 0.827 0.285 0.239 0.786 0.031	
$E_a = -0.398 + 0.535 E_{est}$		10 cm, ste	10 cm, steel	

Equati

Equation 5-26.

Also note the difference in slopes. The much higher slopes for steel ML's would seem to indicate that the model did not overestimate evaporation nearly as much for steel as it did for plastic ML's. Plotting E_a vs. E_{est} on a daily basis revealed the opposite to be true (Figure 5-2). On day 92 there was no discernable difference in $\boldsymbol{E}_{\text{est}}$ between steel and plastic ML's and the range of $E_{\mbox{\scriptsize est}}$ was small. There was slightly more evaporation from steel than from plastic ML's. These facts tended to make the upper ends of regression lines for steel ML's higher than those for plastic. On subsequent days actual evaporation from steel was the same or slightly lower than that from plastic ML's, while $E_{\rm est}$ was always higher for steel ML's. The latter facts tend to make the lower ends of regression lines for steel ML's lower than lines for plastic. Thus the higher slopes for steel ML's were a result of the model overestimating evaporation for steel ML's. Likewise, the negative intercepts for steel ML's were a result of this overestimation, not a result of negative evaporation measurements. Although balance imprecision caused some negative values of evaporation from ML's, the average evaporation on day 100, for example, was still positive.

The difference in slopes is understandable in light of the observation, made in Chapter 3, that steel ML's had significantly lower daytime temperatures than did plastic.

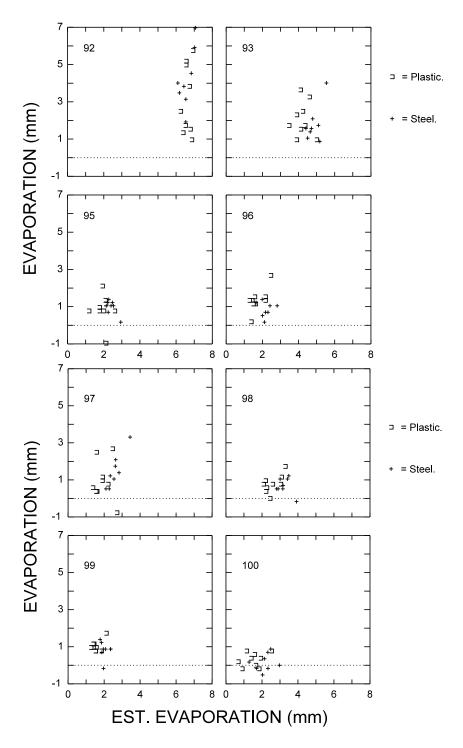


Figure 5-2. Actual evaporation plotted vs. that estimated by the EBM on a daily basis.

Although cumulative evaporation from 20 and 30 cm long steel ML's was lower than that for plastic ML's of the same lengths the difference was significant only for the 20 cm length. Since, for steel ML's, the quantity $(T_{o,max} - T_{d,max})$ would be larger, the model would generate larger estimates of evaporation. Since the lower temperatures of steel ML's were largely due to higher heat fluxes these larger estimates of evaporation could be generated even if actual evaporation were smaller. On the first day after irrigation these differences between steel and plastic ML's were reduced by the fact that the available energy was largely used by evaporation occurring at potential rates. As shown in Figure 4-6, net heat flux was low on the first day after irrigation.

These results indicate that ML length and wall material affect both the actual rate of evaporation from ML's; and, affect the ability of Equation 5-26 to predict evaporation from ML's. If Equation 5-26 were incorporating the effects of all significant physical phenomena then we would not expect to see the markedly different lines which result from the regressions of E_a vs. $E_{\rm est}$. Nor would we expect the lack of correlation on a daily basis between E_a and $E_{\rm est}$ (Figure 5-2, especially day 92). Therefore it is likely that some of the simplifying assumptions, made in the derivation of Equation 5-26, do not hold for the conditions of this study.

Examination of Model Assumptions.

The EBM can be rewritten concisely by substituting Equation 5-25 into 5-26 resulting in:

$$\int E dt = (T_{o,max} - T_{d,max}) S/\mathbf{L_e}$$
 [5-30]

In the previous section it was shown that regression of actual evaporation, E_a , against $(T_{o,max} - T_{d,max})$ resulted in a better fit than regression of E_a against the EBM predictions (R.H.S. of Equation 5-30). Also, the EBM over-estimated evaporation by about 100% on average. Since $\mathbf{L_e}$ is a constant there must be some inaccuracy arising from the assumptions implicit in S. The following assumptions were made during the derivation of Equation 5-26:

1. The diurnal integral of the differences, between a dry and a drying soil, of soil heat flux and of reflected shortwave radiation was assumed neglible compared to the integral of latent heat flux (Equation 5-7):

$$\int [G_o - G_d + K_{in}(\alpha_o - \alpha_d)] dt << \int \mathbf{L_e} E dt$$

This assumption may not hold true considering the soil heat flux values calculated in Chapter 3 for field soil and ML's. Not only were the values of positive heat flux a large fraction of the average evaporation reported here

but there were relatively large differences in heat flux between steel and plastic ML's. Comparisons of heat flux in dry and drying soils will be made later in this chapter.

2. The aerodynamic resistance r, s/m, was described by Equation 5-11:

 $r = 126 U^{-0.96}$

Equation 5-11 was developed for a 50 cm tall sugar beet crop (Rosenberg 1974, p. 83) and is unlikely to hold for bare soil.

- 3. The aerodynamic resistance r was assumed everywhere the same so that H_d could be subtracted from H_o . This appears reasonable for a flat field such as was used in this study but, as will be discussed later, there may exist a problem with using the soil air temperature difference for the reference dry soil in calculating the sensible heat flux, H_o .
- 4. Emissivity was assumed to be constant over a day and was taken equal to 0.95. Since emissivity varies only a few percent as the soil goes from a wet to a dry condition this assumption is unlikely to introduce much error.
- 5. The soil surface temperature was described by a sine wave function (Equation 5-15):

$$T(t) = \overline{T} + 0.5(T_{max} - T_{min}) \sin(wt)$$

For this experiment the measured soil surface temperatures were not sinusoidal on the best of days and on some days had multiple peaks (Figures 3-5 through 3-8).

- 6. The quantity $(T_\circ^4 T_d^4)$ was approximated by $\overline{T}_m^3 (T_\circ T_d)$. This assumption alone introduces little error.
- 7. The diurnal minimum drying soil surface temperature was assumed equal to the minimum dry soil surface temperature resulting in the approximation (Equation 5-21):

$$T_o - T_d = 0.5 (T_{o,max} - T_{d,max}) (1 + sin(wt))$$

During the present study the minimum temperatures of dry and drying soils, as measured by IR thermometry just before dawn, were nearly the same. Therefore this assumption appears reasonable if the fifth assumption is reasonable. However, temperatures measured by thermistor show that the dry soil may be several degrees colder than the field soil during the pre-dawn hours (Figures 3-5 through 3-8).

8. The quantity \overline{T}_m^3 was assumed constant over a 12 hour period. This assumption causes the longwave radiation term to be under-estimated by about 15% when Equation 5-23 is integrated (Appendix B).

- 9. Wind speed was assumed constant during daylight hours leading to the assumption that r was constant during daylight hours. Wind speed obviously was not constant during this Experiment 1 (Figure 3-4).
- 10. All energy flux terms were assumed to be in phase with the soil heat flux which was assumed to be positive from -3 to 9 hours (based on 0 hours occurring when soil temperature is increasing and just equals its average). As shown in Chapter 4, solar and net radiation are likely to be in phase with heat flux at dawn, but heat flux becomes positive at or after dawn, not at 3 hours before dawn (Figure 4-5).
- 11. The latent heat flux $\mathbf{L_e}\mathrm{E}$ was assumed to be neglible except during the 12 hour period during which the soil heat flux was assumed to be positive.

As noted, assumptions numbered 3, 4, 6, and 7 appear reasonable and likely to introduce little error (provided, in the case of assumption 7, that assumption 5 is true). Examination of Equation 5-20 shows two possible reasons for the over-prediction of evaporation. First, the aerodynamic resistance term, r, is a dominant factor influencing the slope of the regression lines in Figures 5-1A and 5-1B. Equation 5-11 (2nd assumption) probably under-predicts r since more

turbulence would be expected to develop within and above a 50 cm tall sugar beet crop than over bare soil. Under-prediction of r would cause over-prediction of E.

The second possible cause of over-estimation lies in the instantaneous temperature depression term, $(T_o - T_d)$. Recall that $(T_o - T_d)$ was replaced by Equation 5-21 which was based sinusoidal on the assumptions of а diurnal surface temperature; and, the equality of minimum daily temperatures in dry and drying soils. Since the minimum value of T_{\circ} may be less than the minimum of $T_{\rm d}$ and since surface temperatures were not sinusoidal (Figures 3-5 - 3-8) it is possible that $(T_o - T_d)$ is overestimated by Equation 5-21 with the result that the EBM overestimates E.

Three possible causes of model inaccuracy arise from the integration of Equation 5-23 over a single 12 hour period to arrive at the EBM. Since half-hourly average wind speed often varied by an order of magnitude during any 24 hours (Figure 3-4) the assumption of constant wind speed (and thus constant r) did not hold. It may prove more accurate to integrate over a half-hour time step using the average measured wind speeds for the shorter period. Second, if half-hourly average measured soil surface temperatures were available then integration with the shorter time step could also increase model accuracy compared to that obtained with the sine wave model of surface temperature. Finally, integration with a

shorter time step would eliminate some of the error caused by assuming that \overline{T}_m^3 was constant over 12 hours (8th assumption).

The next chapter will consider these assumptions in more detail and will demonstrate improvements to the EBM based on changes in, or elimination of assumptions.